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AEOLUS

Algorithmic Principles for Building Efficient Overlay Computers

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Algorithmic solutions for critical resource sharing

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Contents

1 Introduction

This deliverable deals with the results obtained in WP2.2 on Critical Resource Sharing. We mainly present the work done during the second year with a brief summary of the context. More references can be found in the survey done last year [AEO06].

Pursuing the work done in the first year we have obtained new results on WDM networks. A generic problem in the design of optical networks, consists of satisfying a family of requests (or a traffic matrix) under various constraints like capacity constraints. The optimization problem associated consists in designing, for a given family of requests, a network optimizing some criteria, such as minimizing the number of wavelengths or the number of ADMs (Add Drop Multiplexers).

In [Car07], we develop iterative algorithms for resource sharing problems which apply in particular to maximize the number of connections in a Wavelength Division Multiplexing (WDM) optical ring. In [BC07] we determine in particular the minimum number of wavelengths needed in a DAG and introduce a new class of DAG's, those with unique path property.

We also pursued our study of grooming problems [APS07c, APS07b, BBC07] getting the first inapproximability results for the ring or path and constructions for the path.

As indicated in the implementation plan, we focused on fault tolerance issues (see also WP1.4 for other results [AEO07a]). We addressed in [CDP$^+$07, CHPV07] fault tolerance issues in multi-layer networks with shared risk resource groups (i.e. a set of resources breaking down simultaneously when a given failure occurs). We also pursued our work on protection by cycles in [BY07]. In [HD07] we consider reconfiguration in WDM networks in order to capture the dynamicity of the requests and therefore to be nearer from real time problems.

Finally we consider resource sharing in wireless networks. We do not describe here the work on gathering in radio networks, a particular of the so called "call scheduling problem", described in [AEO07b]. We try to consider real-time problems as suggested by the referees last year. For example we consider the famous IEEE 802.11 technology to communicate on the wireless medium. One of these problems is known as the performance anomaly of 802.11 where the presence of slow stations in a multi-rate wireless network slows down all other stations. Several solutions have been proposed in the literature to solve this problem. but they are static or centralized. In [RGLIF07], we tackle both issues, solving the performance anomaly with a dynamic and distributed approach.

In [GH07], we consider the Round Weighting Problem in wireless networks. It solves a joint routing and scheduling problem to attend a given demand subjected to the multi-access interferences. Another approach for gateway placement is done in [GMRR07].

2 Resource scheduling in WDM networks

2.1 Iterative algorithm for resource sharing problems

In [Car07], we study the maxColoring problem defined as follows. We are given an integer w, a set V and a set $\mathcal{S}$ of subsets of V called *compatible sets* ($\mathcal{S}$ is closed under subsets). The objective is to compute a subset of w disjoint sets of $\mathcal{S}$ whose union contains a maximum number of elements of V. In other words, we are seeking for an assignment of colors (to be thought of as resources) to as many elements of V as possible so that at most w different colors are used and, for each color, the set of elements colored with this color is a compatible set. The compatible sets can be given either explicitly or implicitly. For example, for $w = 1$ and by defining the compatible sets to be the independent sets of a graph, the problem is identical to the maximum independent set problem.

The maxColoring problem is strongly related to the problem of computing a compatible set of maximum size. A simple iterative algorithm repeatedly (i.e., w times) includes a compatible set of maximum size that does not contain elements that are contained in compatible sets selected before. Awerbuch et al. [AAF+01] (see also [WL98]) have shown that, using an algorithm that computes a compatible set of size at most ρ times smaller than the size of the maximum compatible set, the corresponding iterative algorithm has approximation ratio at least $\frac{1}{1-\exp(-1/\rho)}$. Even in the case where a compatible set of maximum size can be computed in polynomial time (this is trivial if all compatible sets are given explicitly), the approximation ratio of the iterative algorithm is $\frac{e}{e-1} \approx 1.58198$. In general, this bound is best possible. A maxColoring algorithm with strictly better approximation ratio could be executed repeatedly to approximate minimum set cover within a factor of $\alpha \ln n$ for some constant $\alpha < 1$, contradicting a famous inapproximability result due to Feige [Fei98].

In [Car07] we are interested in solutions of instances of the maxColoring problem when a compatible set of maximum size can be computed in polynomial time. We study the class of iterative maxColoring algorithms which try to accommodate elements of V by computing as many as possible disjoint compatible sets of the maximum size. This involves solving instances of the *k-set packing problem*. An instance of k-set packing consists of a set of elements V, a set $\mathcal{S}$ of subsets of V each containing exactly k elements, and the objective is to compute a maximum number of disjoint elements of $\mathcal{S}$. A solution to this problem is called a k-set packing. A k-set packing is called *maximal* if it cannot be augmented by including another set of $\mathcal{S}$ without loosing feasibility. An iterative maxColoring algorithm works as follows:

Input: An integer w, a set V and a set of compatible sets $\mathcal{S} \subseteq 2^V$.
Output: At most w disjoint sets T_1, T_2, ..., of $\mathcal{S}$.

1. Set $F := V$, $\mathcal{T} := \mathcal{S}$, $i := 1$ and denote by k the size of the largest compatible set in $\mathcal{T}$.

2. While $i \leq w$ or $F \neq \emptyset$ do:

   (a) Compute a maximal k-set packing Π among the sets of $\mathcal{T}$ of cardinality k.

   (b) If $\Pi \neq \emptyset$ then

      i. Denote by $I_0, I_1, ..., I_{t-1}$ the compatible sets in Π and set $F' := \cup_{j=0}^{\min\{w-i,t-1\}} I_j$.

      ii. For $j := 0, ..., \min\{w-i, t-1\}$, set $T_{i+j} := I_j$.

      iii. Set $F := F \setminus F'$, $\mathcal{T} := \mathcal{T} \setminus \cup_{S \in \mathcal{T}: F' \cap S \neq \emptyset} S$ and $i := i + t$.

   (c) Set $k := k - 1$.

The algorithm that iteratively computes a compatible set of maximum size (henceforth called the *basic iterative algorithm*) can be thought of as an algorithm belonging to the above class of algorithms. In step 2a, it computes a maximal k-set packing by iteratively computing compatible sets of size k and removing from F the elements in the compatible sets computed. Since including a compatible set of size k may force at most k compatible sets of an optimal k-set packing to be excluded from the solution, this algorithm has approximation ratio k for solving the k-set packing problem. Using different methods for computing k-set packings, we obtain different algorithms. When the maximum compatible set has constant size κ, computing a maximal k-set packing can be done using a local search algorithm. Consider the set S of all compatible sets of V of size κ. A local search algorithm uses a constant parameter p (informally, this is an upper bound on the number of local improvements performed at each step) and, starting with an empty packing Π, repeatedly updates Π by replacing any set of $s < p$ sets of Π with $s + 1$ sets so that feasibility is maintained and until no replacement is possible. This algorithm is analyzed in [HS89] where it is proved that its approximation ratio is $\frac{k}{2} + \epsilon$, where ϵ can become arbitrarily small by using large (but still constant) p.

We have developed a *benefit-revealing LP lemma* which relates the approximation ratio of iterative algorithms with the approximation ratio of the k-set packing algorithms used in step 2a. This lemma can be extremely helpful for the analysis of the performance of iterative algorithms on instances of maxColoring where a maximum compatible set can be computed in polynomial time and, additionally, the ratio $|\mathcal{OPT}|/w$ is upper-bounded by a (small) constant, where $\mathcal{OPT}$ denotes an optimal solution. For these instances, the $\frac{e}{e-1}$ bound for the basic iterative algorithm following by the analysis in [AAF$^+$01, WL98] can be improved. The new proofs are not particularly complicated and they require solving a few simple linear programs. Consider the problem where we are given a graph G in which the size of the maximum independent set is bounded by a constant κ and an integer w and we aim to compute a w-colorable subset of nodes of maximum cardinality. This is a maxColoring problem defined over the independent sets of G. This problem can be easily proved to be APX-hard even when $\kappa = 3$ through a simple approximation-preserving reduction from 3-dimensional matching [Kan91a]. Using local search algorithms for k-set packing as subroutines, we obtain iterative maxColoring algorithms for which the analysis with the benefit-revealing LP yields improved upper bounds on their approximation ratio.

This approximation ratio is 9/7 for $\kappa = 3$ and stays always smaller than 3/2 for all values of $\kappa \leq 18$.

In [Car07], we specifically apply the technique to the resource sharing problem of maximizing the number of connections in a Wavelength Division Multiplexing (WDM) optical ring. In this case, the ratio of the size of the optimal solution over the number of available wavelengths is not bounded in general. Hopefully, very simple algorithms are efficient when this ratio is large while iterative algorithms are proved to be efficient for small values of this ratio through the benefit-revealing LP analysis. So, all the algorithms considered in [Car07] have the same structure. They execute a very simple algorithm called CL and an iterative algorithm on the input instance and output the best among the two solutions. This approach leads to new approximation algorithms for the problem which improve previous results in the literature [NPZ03b, NPZ03a].

2.2 Minimum number of wavelengths in a DAG

The problem we consider is motivated by routing, wavelength assignment and grooming in optical networks. But it can be of interest for other applications in parallel computing, where the graph will represent for example the precedence graph of a program or for scheduling complex operations on pipelined operators. A generic problem in the design of optical networks, consists of satisfying a family of requests (or a traffic matrix) under various constraints like capacity constraints. The optimization problem associated consists in designing, for a given family of requests, a network optimizing some criteria, such as minimizing the number of wavelengths or the number of ADMs (Add Drop Multiplexers).

A request is satisfied by assigning to it a dipath in the network. A family of requests is satisfied, if we can route them in such a way that the capacity constraints of the network are satisfied. This is known as the routing problem. For a given routing let us define the load of an arc as the number of routes (dipaths) containing it and the load of the routing as the maximum load of the arcs. Typically one wants either to insure that the load of an arc does not exceed the capacity of this arc or to minimize the load of a routing satisfying a given family of requests.

Note that requests are satisfied on a virtual (logical) network which is itself embedded in the physical network (in fact there might be many layers). It is the case for example when considering SONET/WDM rings or in MPLS over WDM networks; in the latter case the RWA problem has to be considered for the lightpaths [DR00, DR02b]. Anyway, at the conceptual level of modeling of this article, the problems are the same and we will use the word request to indicate a connection at the upper level.

Minimizing the load or/and the number of wavelengths is a difficult problem and in general an NP-hard problem. These problems have been extensively studied in the literature for various topologies or special families of requests like multicast or all-to-all (see for example the survey [BBG$^+$97] or [KK99, RS95]). Many particular cases where the minimum number of wavelengths is equal to the minimum routing load have been given.

For example, in [BHP98] it is shown that for any digraph and for a multicast instance (all the requests have the same origin), there is equality and both problems can be solved in polynomial time. For some topologies the load might be easily computed, but the minimum number of wavelengths is NP-hard to compute as it is related to coloring problems. This is the case for symmetric trees (see the survey [CKP01]). However, for symmetric trees it has been proved that there is equality for the all to all instance ([GHP01]) and approximation algorithms have been given ([CKP01, EJK+99]).

As the RWA problem is very difficult to solve, it is often split into two separate problems. First one solves the routing problem by determining dipaths which minimize the load or are easy to compute like shortest paths. Then, the routing being given, the wavelength assignment problem is solved. In that case the input of the problem is not a family of requests but a family of dipaths $\mathscr{P}$. We will denote by $\pi(G,\mathscr{P})$ the maximum of the load of all the arcs of the digraph G for the family $\mathscr{P}$. Determining the minimum number $w(G,\mathscr{P})$ of wavelengths (colors) needed to color a family of dipaths $\mathscr{P}$ in such a way that two dipaths with the same wavelength are arc-disjoint is still NP-hard in that case. Indeed it corresponds to finding the chromatic number of the conflict graph whose vertices represent the dipaths and where two vertices are joined if the corresponding dipaths intersect.

There are examples of topologies where there are at most 2 dipaths using an arc ($\pi(G,\mathscr{P}) = 2$), but where we need as many wavelengths as we want (see Figure 1 for the example for 4 wavelengths) and so for these digraphs there is no ratio between $w(G,\mathscr{P})$ and $\pi(G,\mathscr{P})$. In the example we consider k dipaths from s_i to t_i. The dipaths starts in s_i, then go alternatively right and down till they arrive at the bottom where they go right and up till they arrive at the destination t_i. Any two dipaths intersect so the conflict graph is complete and we need k colors. However the load of an arc is at most 2.

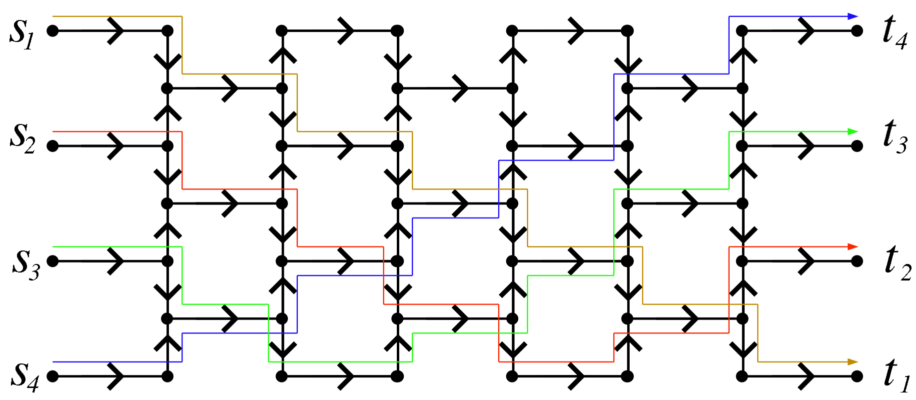

Figure 1: A pathological example

Note that, if the original digraph has the property that for any request (x, y) there is a unique dipath from x to y, then it is equivalent to consider a family of requests or a family of dipaths.

In [BC07] we consider the class of Directed Acyclic Graphs, DAGs, which plays a central role in Parallel and Distributed Computing. Part of our motivation came when we tried to extend the results obtained in [BCCP06] for paths motivated by grooming problems for the paths ([BBC07, DR02b]). In fact, we first proved that for rooted trees (directed trees where there is a unique dipath from the root to any vertex), for any family of requests, the minimum number of wavelengths is equal to the load.

The example given above being a DAG there is no hope to bound ratio between $w(G,\mathscr{P})$ and $\pi(G,\mathscr{P})$. In this paper we fully characterize when $w(G,\mathscr{P}) = \pi(G,\mathscr{P})$ for a DAG. In fact the necessary and sufficient condition is that G does not contain what we call an internal cycle, i.e. an oriented cycle, such that all the vertices have at least one predecessor and one successor in G (said otherwise all cycles contain neither a source nor a sink).

We also consider an apparently new class of DAGs, which is of interest in itself, those for which there is at most one dipath from a vertex to another. We call this property the UPP (Unique diPath Property) and call these digraphs UPP-DAGs. For these UPP-DAGs we show that the load is equal to the maximum size of a clique of the conflict graph. We show that if an UPP-DAG has only one internal cycle, then for any family of dipaths $w(G,\mathscr{P}) = \left\lceil \frac{4}{3}\pi(G,\mathscr{P})\right\rceil$ and we exhibit an UPP-DAG and a family of dipaths reaching the bound. We conjectured that the ratio between $w(G,\mathscr{P})$ and $\pi(G,\mathscr{P})$ cannot be bounded. This conjecture has been proved recently.

The DAGs of the pathological examples have many internal cycles. In Figure 2 , we give an example of a DAG with one internal cycle and a set of 5 dipaths $\mathscr{P}$ such that $\pi(G,\mathscr{P}) = 2$ and $w(G,\mathscr{P}) = 3$. The dipaths are a_1, b_1, c_1 ; b_1, c_1, d_1 ; c_1, d_1, e_1; b_1, d_1, e_1 via the second dipath from b_1 to d_1; a_1, b_1, d_1 also via this second dipath. The load is 2 and the conflict graph is a cycle of length 5 and so we need 3 colors to color its vertices. Another examples of family of dipaths $\mathscr{P}$ with $\pi(G,\mathscr{P}) = 2$ and $w(G,\mathscr{P}) = 3$ in an UPP-DAG is given in section 4 Figure 3 and gives rise to an infinite family attaining the bound of $\frac{4}{3}$.

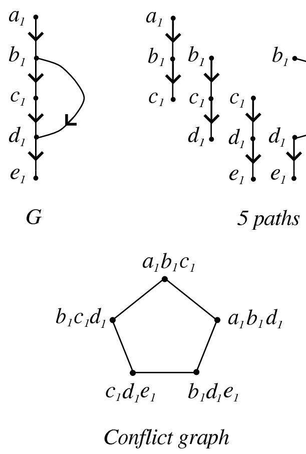

Figure 2: Example for a DAG with an internal cycle

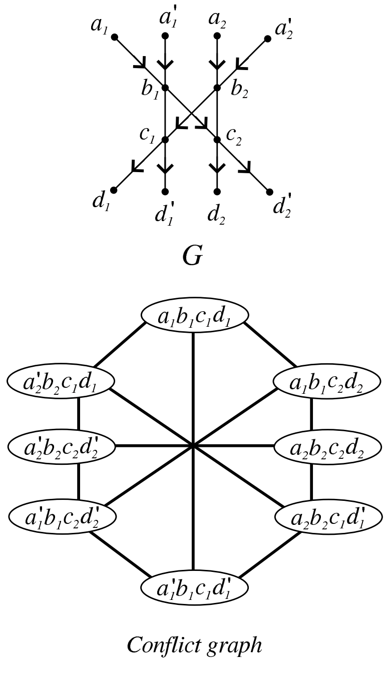

Figure 3: Example for an UPP-DAG

2.3 Traffic grooming

Traffic grooming is the generic term for packing low rate signals into higher speed streams (see the surveys [BC06, DR02b, ML01, Som01, ZM03]). By using traffic grooming, one can bypass the electronics in the nodes which are not sources or destinations of traffic, and therefore reduce the cost of the network. Typically, in a WDM (Wavelength Division Multiplexing) network, instead of having one SONET Add Drop Multiplexer (ADM) on every wavelength at every node, it may be possible to have ADMs only for the wavelengths used at that node (the other wavelengths being optically routed without electronic switching).

SONET ring is the most widely used optical network infrastructure today. In these networks, a communication between a pair of nodes is done via a *lightpath*, and each lightpath uses an Add-Drop Multiplexer (*ADM*), i.e. an electronic termination, at each of its two endpoints. If each request uses $\frac{1}{C}$ of the capacity of a wavelength, C is said to be the *grooming factor*. The problem is equivalent to assigning a wavelength to each request in such a way that for any wavelength and any link of the network, there can be at most C requests using this link on this wavelength. The aim is to minimize the total number of ADMs. In a graph-theoretical approach, the set of requests is modeled by a graph R; to each wavelength is associated a subgraph and each vertex in the subgraph of R corresponding to a wavelength represents an ADM used for this wavelength. The problem, in the case where the communication network is a ring, can be formally stated as follows:

Ring Traffic Grooming

Input: A cycle C_n on n vertices (network), a graph R (set of requests) on vertices of C_n, and a grooming factor C.

Output: Find for each edge $r = \{x, y\}$ of R, a path $P(r)$ in C_n between x and y, and a partition of the edges of R into subgraphs R_ω, $1 \leq \omega \leq W$, such that for each edge e in $E(C_n)$ and for all ω, the number of paths $P(r)$ using e, r being an edge of R_ω, is at most C.

Objective: Minimize $\sum_{\omega=1}^{W} |V(R_\omega)|$.

The statement of Path Traffic Grooming is analogous, replacing C_n by P_n and also it is the same for a general graph G.

To fix ideas, consider a ring on five nodes and the complete graph of Figure 4 as request graph, and let $C = 2$. We exhibit two valid solutions of the problem, both using two subgraphs (i.e. two wavelengths). The second solution is better because it uses 9 vertices instead of 10.

2.3.1 Hardness and Approximation of Traffic Grooming in Rings and Paths

Let us briefly summarize the state-of-the-art of the complexity of traffic grooming. It has been proved to be NP-complete for ring networks and general C [CM00]. On the

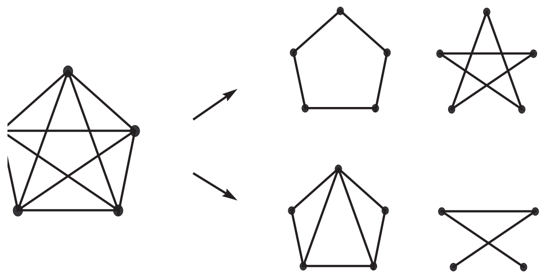

Figure 4: Two valid partitions of K_5 when $C = 2$, using different number of ADMs

other hand, there was no result on the inapproximability of the problem for fixed $C \geq 1$. In [CL04] the authors conjectured that TRAFFIC GROOMING was MAX SNP-hard (or equivalently, APX-hard, modulo PTAS-reductions) for any fixed value of the grooming factor. We have answered affirmatively to this question in [APS07c], providing the first hardness result for the RING TRAFFIC GROOMING problem for fixed values of the grooming factor C.

Considering C as part of the input, in [HDR06] it was proved that PATH TRAFFIC GROOMING does not accept a constant-factor approximation unless P = NP. For fixed values of C, PATH TRAFFIC GROOMING was proved to be in P for $C = 1$ [BC06], but the complexity for fixed $C \geq 2$ has been an open question for a while. Recently, it has been proved in [SUZ07] that PATH TRAFFIC GROOMING for fixed $C > 1$ is NP-complete for *bounded number of wavelengths*. Our method permits us to improve this result, by proving the APX-completeness of PATH TRAFFIC GROOMING for any fixed $C > 1$ and unbounded number of wavelengths [APS07c]. In particular, this extends the NP-completeness result of [SUZ07] to the case where the number of wavelengths is not bounded.

The main ingredient of our approach is the proof of the APX-completeness of the problem of finding the maximum number of edge-disjoint triangles in a graph with bounded degree B: MAXIMUM BOUNDED EDGE COVERING BY TRIANGLES (MECT-B for short). The proof is obtained by L-reduction from MAXIMUM BOUNDED COVERING BY 3-SETS, which was proved to be MAX SNP-complete in [Kan91b]. A simple modification of this technique permits us to prove the APX-completeness of finding the maximum number of edge-disjoint odd cycles of given length in a graph. This later claim is then used to extend our results to arbitrary values of C, see [APS07a] for an extended version including the appendices.

The design of approximation algorithms for TRAFFIC GROOMING is the topic of the second part of [APS07c]. As we discuss in [APS07c], it is trivial to obtain a $\mathcal{O}(\sqrt{C})$-approximation with running time polynomial in C and n. For $C = 1$, the best algorithm in rings achieves an approximation ratio of 10/7 [EL04]. For general C, the best approximation algorithm [FMSZ05] achieves an approximation factor of $\mathcal{O}(\log C)$, but the problem is that the running time is exponential in C (that is, n^C). Since in practical applications SONET WDM rings are widely used as backbone optical networks [DR02b], [ML01], the grooming factor is usually greater than the size of the network, i.e. $C \geq n$. For those networks, the running time of this algorithm becomes exponential in n. Thus, it turns out to be important to find good approximation algorithms with running time polynomial in both n and C. In [APS07c] we have provided such an approximation algorithm, considering C as part of the input. Our algorithm finds a solution of Ring Traffic Grooming that approximates the optimal value within a factor $\mathcal{O}(n^{1/3}\log^2 n)$ for any $C \geq 1$. To the best of our knowledge, this is the first polynomial-time approximation algorithm for the Ring Traffic Grooming problem with an approximation ratio which does not depend on C. Although the performance of this algorithm seems not to be very good at first sight, in fact we conjecture that for the general instance of the problem it is not possible to get rid of a factor n^δ, for some constant $\delta > 0$. Finally, we have showed that the general scheme of the algorithm yields a $\mathcal{O}(\log^2 n)$-approximation if the request graph excludes a fixed graph as minor, for example if R is planar or of bounded genus. The main theoretical contribution of the second part of the paper is to relate the Traffic Grooming problem to the Dense k-Subgraph problem [FPK01]. A preliminary shorter version of [APS07c] appeared first in [APS07b]. The main results can be summarized as:

Theorem 2.1 ([APS07c]) Ring Traffic Grooming *is* APX-*complete for all fixed* $C \geq 1$. Path Traffic Grooming *is* APX-*complete for any fixed* $C \geq 2$. *Thus, they do not accept a* PTAS *unless* P = NP.

Theorem 2.2 ([APS07c]) *There exists a polynomial-time approximation algorithm that approximates* Ring Traffic Grooming *within a factor* $\mathcal{O}(n^{1/3}\log^2 n)$ *for any* $C \geq 1$.

2.3.2 Traffic Grooming in Unidirectional Rings with Two Periods of Traffic

If we consider unidirectional SONET/WDM ring networks, then the routing is unique and we have to assign to each request between two nodes a wavelength and some bandwidth on this wavelength. Furthermore if the traffic requirement is symmetric, it can be easily shown (by exchanging wavelengths) that there always exists an optimal solution in which the same wavelength is given to a pair of symmetric requests. Then without loss of generality we will assign to each pair of symmetric requests, called a *circle*, the same wavelength. Then each circle uses $\frac{1}{C}$ of the bandwidth in the whole ring.

In [BS07] we have studied the problem for a unidirectional SONET ring with n nodes, a grooming ratio C, and an all-to-all uniform unitary traffic. This problem has been modeled as a graph partition problem in both [BC03a] and [GHLO03]. In the all-to-all case the set of requests is modeled by the complete graph K_n. To a wavelength λ is associated a subgraph B_λ in which each edge corresponds to a pair of symmetric requests (that is, a circle) and each node to an ADM. The grooming constraint, i.e. the fact that a wavelength can carry at most C requests, corresponds to the fact that the number of edges $|E(B_\lambda)|$ of each subgraph B_λ is at most C. The cost corresponds to the total number of vertices used in the subgraphs:

Traffic Grooming in Unidirectional Rings

Input: Two integers n and C.

Output: Partition $E(K_n)$ into subgraphs B_λ, $1 \le \lambda \le \Lambda$, s.t. $|E(B_\lambda)| \le C$ for all λ.

Objective: Minimize $\sum_{\lambda=1}^{\Lambda} |V(B_\lambda)|$.

This problem has been well studied when the network is a unidirectional ring [BC03b, BCC+05, BCLY04, BC03a, BC06, BCM03, CM00, DR02a, DR02b, GHLO03, GLS98, GRS00, Hu02a, Hu02b, ML01, WCLF00, WCVM01, YF02, ZQ00]. With the all-to-all set of requests, optimal constructions for a given grooming ratio C were obtained using tools of graph and design theory [CD06, DS92], in particular for grooming ratio $C = 3$ [BC03b], $C = 4$ [Hu02a, BCM03], $C = 5$ [BCLY04], $C = 6$ [BCC+05] and $C \ge N(N-1)/6$ [BCM03].

Most of theses papers deal with a single (static) traffic matrix. Some articles consider the case of variable (dynamic) traffic, like finding a solution which works for the maximum traffic demand [BM00b, ZZM03], but all keep a fixed grooming factor. In [CQS07] an interesting variation of the traffic grooming problem has been introduced in order to capture some dynamicity of the traffic : the grooming for two-period optical networks. Informally, in the two-period grooming problem each time period supports different traffic requirements. During the first period of time there is an all-to-all uniform traffic between n nodes, each request using $1/C$ of the bandwidth; but during the second period there is an all-to-all traffic only between a subset V of v nodes, each request being now allowed to use a larger fraction of the bandwidth, namely $1/C'$ where $C' < C$. Denote by X the subset of n nodes. Therefore the two-period grooming problem can be expressed as follows:

Two-Period Grooming in Unidirectional Rings

Input: Four integers n, v, C, and C'.

Output: A partition of $E(K_n)$ into subgraphs B_λ, $1 \le \lambda \le \Lambda$, such that for all λ, $|E(B_\lambda)| \le C$, and $|E(B_\lambda) \cap (V \times V)| \le C'$, with $V \subseteq X$, $|V| = v$.

Objective: Minimize $\sum_{\lambda=1}^{\Lambda} |V(B_\lambda)|$.

Following [CQS07], a grooming is denoted by $N(n, C)$. When the grooming $N(n, C)$ is *optimal*, i.e. minimizes the total ADM cost, then the grooming is denoted by $\mathscr{ON}(n, C)$. Whether general or optimal, the drop cost of a grooming is denoted by *cost* $N(n, C)$ or *cost* $\mathscr{ON}(n, C)$, respectively.

A grooming of a two-period network $N(n,v;C,C')$ with grooming ratios (C,C') coincides with a graph decomposition $(X,\mathcal{B})$ of K_n (using standard design theory terminology, $\mathcal{B}$ is set of all the *blocks* of the decomposition) such that $(X,\mathcal{B})$ is a grooming $N(n,C)$ in the first time period, and $(X,\mathcal{B})$ faithfully embeds a graph decomposition of K_v such that $(V,\mathcal{D})$ is a grooming $N(v,C')$ in the second time period. We use the notation $\mathscr{ON}(n,v;C,C')$ to denote an optimal grooming $N(n,v;C,C')$.

As it turns out, an $\mathscr{ON}(n,v;C,C')$ does not always coincide with an $\mathscr{ON}(n,C)$. Generally we have *cost* $\mathscr{ON}(n,v;C,C') \geq$ *cost* $\mathscr{ON}(n,C)$. Of particular interest is the case when *cost* $\mathscr{ON}(n,v;C,C') =$ *cost* $\mathscr{ON}(n,C)$.

C.J. Colbourn, G. Quattrochi and V.R. Syrotiuk solved completely in [CQS07] the case $C=2$ and $C=3$ ($C'=1$ or 2). We have solved in [BS07] the case $C=4$, that is $(C,C')=(4,1)$, $(4,2)$, and $(4,3)$. We summarize below the cost formulas we have obtained for $n>4$:

Theorem 2.3 ([BS07])

- $$\textit{cost}\ \mathscr{ON}(n,v;4,1) = \begin{cases} \binom{n}{2} & \textit{if } v \leq \lceil \frac{n}{2} \rceil \\ \binom{n}{2} + \binom{v}{2} - \left\lfloor \frac{v(n-v)}{2} \right\rfloor & \textit{if } v > \lfloor \frac{n}{2} \rfloor \end{cases}$$

- $$\textit{cost}\ \mathscr{ON}(n,v;4,2) = \begin{cases} \binom{n}{2} & \textit{if } v \leq \lfloor \frac{2n}{3} \rfloor \\ \binom{n}{2} + \left\lceil \frac{\binom{v}{2} - v(n-v)}{2} \right\rceil + \varepsilon & \textit{if } v > \lfloor \frac{2n}{3} \rfloor \textit{ and } v \textit{ even} \\ \qquad \textit{where } \varepsilon = \begin{cases} 1 & \textit{if } n-v=2,4 \\ 0 & \textit{otherwise} \end{cases} & \\ \binom{n}{2} + \left\lceil \frac{\binom{v}{2} - (v-1)(n-v) - \lfloor \frac{n-v}{2} \rfloor}{2} \right\rceil + \varepsilon' & \textit{if } v > \lfloor \frac{2n}{3} \rfloor \textit{ and } v \textit{ odd} \\ \qquad \textit{where } \varepsilon' = \begin{cases} 1 & \textit{if } n-v=3 \\ & \textit{and } v \equiv 3 \pmod 4 \\ 0 & \textit{otherwise} \end{cases} & \end{cases}$$

- *cost* $\mathscr{ON}(n,v;4,3) = \binom{n}{2}$

2.3.3 Traffic Grooming in Unidirectional Paths

The traffic grooming problem has also been considered when the network is reduced to a path. In particular, it has been shown in [HDR06] that the grooming problem is NP-complete when the grooming factor is larger that 2, and a polynomial time algorithm for the special case of grooming factor 1 and general set of requests has been given in [BBC07].

Notice that when the network is a path, the shortest path from node i to node j is unique, and we can split the requests into two classes, those with $i < j$ and those with $i > j$. Therefore the grooming problem for P_n^*, that is a bidirectional path, can be reduced to two distinct problems on $\vec{P}_n$, the direct path. In particular we have $A(P_n^*, K_n^*, C) = 2A(\vec{P}_n, TT_n, C)$, where TT_n is a transitive tournament on n vertices, that is the digraph with arcs $\{(i,j) \mid 1 \leq i < j \leq n\}$. We denote $\{a,b,c\}$ the TT_3 with arcs $\{a,b\}$, $\{b,c\}$, and $\{a,c\}$.

For example, $A(\vec{P}_7, TT_7, 2) = 20$, and the partition consists of 6 subgraphs, the 5 TT_5 {2,4,5}, {3,4,6}, {1,5,6}, {2,6,7}, and {1,4,7}, plus the union of two TT_3 {1,2,3}+{3,5,7}.

Theorem 2.4 ([BBC07]) *When n is odd, $A(\vec{P}_n, TT_n, 2) = \lceil (11n^2 - 8n - 3)/24 \rceil$; When n is even, $A(\vec{P}_n, K_n, 2) = (11n^2 - 4n)/24 + \varepsilon(n)$, where $\varepsilon(n) = 1/2$ when $n \equiv 2, 6 \pmod{12}$, $\varepsilon(n) = 1/3$ when $n \equiv 4 \pmod{12}$, $\varepsilon(n) = 5/6$ when $n \equiv 10 \pmod{12}$, and $\varepsilon(n) = 0$ when $n \equiv 0, 8 \pmod{12}$.*

2.3.4 Traffic Grooming in Bidirectional Rings

In a bidirectional ring, requests are routed either clockwise ↻ or counterclockwise ↺. This case has been much less studied than the unidirectional ring, due to its higher complexity. There is an important work providing heuristics for the ring grooming [BM00b, CM00, DR00, GLS98, GLS99, DR01, Som01], but there was still an important lack of theoretical analysis of the problem. In [CL04] a new lower bound was proved for the bidirectional ring (regardless of routing).

In [BCMS06, BMS07] we have focused on a bidirectional ring with symmetric shortest path routing, and on the all-to-all case. The main contributions are the following: we have formally stated the problem in terms of graph partitioning and graph embeddings, which had never been done before in bidirectional rings. Then we have provided lower bounds that improve those existing in the literature:

Theorem 2.5 ([BCMS06, BMS07]) *The number $A(C, n)$ of ADMs required in a bidirectional ring with n nodes and grooming factor C is lower bounded by the expression*

$$A(C, n) \geq \left\lceil \frac{n(n-1)}{2} \frac{k+1}{k(k+1)+r} \right\rceil$$

Where $C = 1 + 2 + \ldots + (k-1) + k + r = \frac{k(k+1)}{2} + r$, with $r < k+1$.

The remainder of the article is devoted to find families of solutions for certain values of C and n. Namely, we have studied the case $C = 2$, improving the general lower bound and providing a $\frac{12}{11}$-approximation for all odd values of n. Then we tackle the case $C = 3$, improving the lower bound when $n \equiv 3 \pmod 4$ and giving optimal solutions when $n \equiv 1, 5 \pmod{12}$:

Theorem 2.6 ([BCMS06, BMS07]) *If $C = 3$, and $n \equiv 1, 5 \pmod{12}$,*

$$A(3, n) = \frac{n(n-1)}{4},$$

and therefore the general lower bound of Theorem 2.5 is achieved.

For all other odd values of n we give asymptotically optimal solutions. Finally we use design theory to provide optimal solutions when C is of the form $1 + 2 + \ldots + k$, for some congruence classes of values of n. The main results are:

Theorem 2.7 ([BCMS06, BMS07]) *If there exists a BIBD(v, k, 1), then there exists an optimal construction for $n = 2v - 1$ and $C = \frac{k(k+1)}{2}$, except possibly for a finite set of values of n.*

This results allow us to find optimal solutions for the following values of C and n:

k	C	n
1	1	All values
2	3	$n \equiv 1, 5 \pmod{12}$
3	6	$n \equiv 1, 7 \pmod{24}$
4	10	$n \equiv 1, 9 \pmod{40}$
5	15	$n \equiv 1, 9 \pmod{30}$
6	21	$n \equiv 1, 13 \pmod{84}$
7	28	$n \equiv 1, 15 \pmod{112}$
8	36	$n \equiv 1, 17 \pmod{144}$

We conclude [BMS07] with a comparison of what is known about exact solutions in unidirectional and bidirectional rings.

3 Fault tolerance

3.1 Shared risk resource group

Modern day networks are multilayer networks where traffic flows are routed on meshed topologies whose links are indeed end-to-end paths on a high bandwidth infrastructure. IP/WDM, MPLS or GMPLS networks, as well as P2P or GRID computing overlay structures are example of such a hierarchy. In these settings, customers expect an uninterrupted service, even in the event of failures such as power outages, equipment failures, natural disasters and cable cuts. Many layer wide protection schemes have been proposed in the literature, as for instance for the WDM optical networks [TR04b, TR04a] or for GMPLS networks [UCM05]. One method to provide survivability is through path protection schemes, in which a disjoint backup path is pre-computed for every working path. Such protection schemes provide 100% reliability against any single link failure in the network layer that is considered. However, in multilayer network settings, a single failure event in the underlying layer may result in the failure of several links in the virtual network. For example, in a MPLS/WDM network, several apparently independent label switched paths (LSP) may be routed on the same fiber. Even though these LSPs represent disjoint links of the MPLS virtual topology, the cut of an optical fiber can cause all the LSPs to fail. For example, in Figure 5 the failure of the link FG induces multiple failures on the set of links of the virtual topology. The concept of *Shared Risk Link Group*, generalized to *Shared Risk Resource Group* [CST01, PPJ$^+$01, SYHG01], modeled correlation between resources. A SRRG is a set of resources breaking down simultaneously when a given failure occurs.

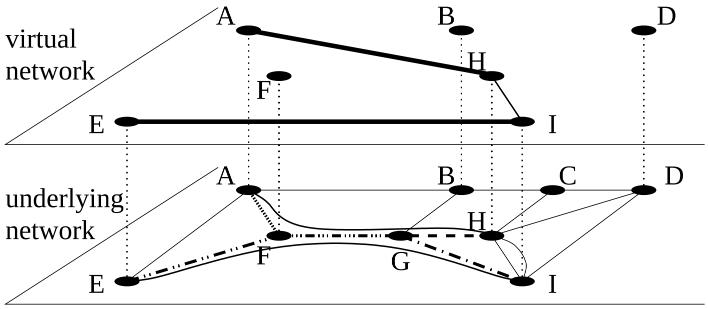

Figure 5: A Shared Risk Link Group in a multilayer networks : AH and EI links share the same risk of failure, FG.

When the risk of failure concerns nodes instead of links the group is said to be a Shared Risk Node Group, and more generally the notion of Shared Risk Resource Group (SRRG) has been defined to encompass any kind of resources sharing a common risk of unavailability. Recently in [YVJ05, CDP$^+$07] the SRRG notion has been formalized through colored graphs: Each SRRG is associated to a color and each link of the network is assigned colors according to which SRRG it belongs. Shared Risk Resource Groups are naturally modeled by associating to each group (or risk) a *color*, and to each edge (or resource) the colors representing the risks affecting it [Far06, YVJ05]. According to the general case of Shared Risk Resource Groups, an edge may belong to several colors, modeling the fact that a resource may face different and independent risks. In [Far06], a *colored graph* is thus defined as an undirected graph associated to a collection of subsets of edges, the colors, covering the edge set. However, depending on the context, this property may be interpreted in several ways whether *all the colors associated to an edge* or *only one color among the set of associated colors* is considered to be used when this edge belongs to a path or a cut or any kind of structure. That is why another definition for colored graphs is presented in [YVJ05]. In case a single color of an edge is used when this edge is crossed by a path, an edge belonging to X colors can easily be replaced by X monochromatic parallel edges, and in case all the colors are used when the edge is crossed, the edge is replaced by a chain of X monochromatic edges. Therefore, we may assume that edges are monochromatic, in other words colors not only cover the edge set but also partition

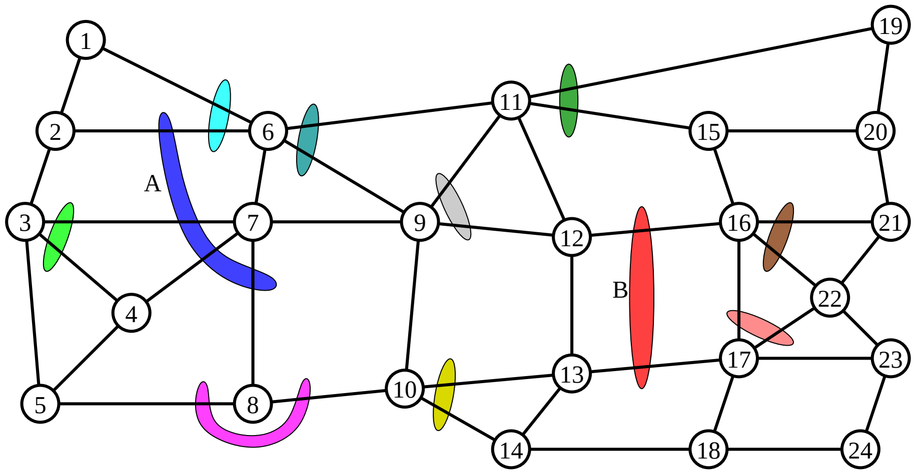

Figure 6: Classical benchmark network for SRRGs problems

it. This definition is actually encompassed in the definition of [Far06], consequently all the complexity and inapproximability results proved with the monochromatic assumption extend to the more general settings of [Far06].

In this context, optimization problems in colored graphs have been studied to answer survivability challenges in the SRRG context [Bha99, CDRV05, Vog06a, Vog06b, CDP+07]. The *span* of a color, that is the number of connected components of the subgraph induced by a the edges of this color, appears to be a very important parameter as the complexity and approximability properties depends on the maximum span of the graph. State-of-the-art results are summarized in Table 1.

3.1.1 Minimum Color *st*-path

As reported in Table 1, the problem of determining a color path from a source s to a destination t is in general NP-complete and hard to approximate. However, it as been shown in [DS04] that this problem can be solved in polynomial time when all colors are of span 1. More interestingly, Theorem 3.1 that generalized the results of [DS04] has been proved in [CPRV06]:

Theorem 3.1 ([CPRV06]) *When the number of colors of span larger than 1 is bounded, the problem of determining a minimum color st-path can be solved in polynomial time.*

The network of Figure 6 has frequently been used to benchmark heuristic algorithms [SYR05, TR04b, TR04a]. However, with a careful analysis of the inputs (network and SRGs) and thanks to Theorem 3.1, we can conclude that the minimum color *st*-path problem can be solved in polynomial time in this case. Indeed, only SRGs A and B have span larger than 1.

Min. Color ···		st-Path	st / Multi -Cut	Cut	2-Disjoint st-Paths	2-Min Overlap. st-Paths	Max. number of Disjoint st-Paths	Spanning Tree
general	complexity	NP[YVJ05]	NP	?	NP-Complete [Hu03]	NP[YVJ05]	NP[JRN04]	NP[CL97]
	non approx	$2^{\log^{1-\delta}\|\mathcal{C}\|^{\frac{1}{2}}}$	$2^{\log^{1-\delta}\|\mathcal{C}\|^{\frac{1}{2}}}$	?	———	?	$\forall\varepsilon > 0,\ \|V\|^{\frac{1}{4}-\varepsilon}$	$o(\log(\|V\|))$ [WCX02]
	approx	?	?	?	———	?	?	$O(\log(\|V\|))$ [WCX02]
span k	complexity	NP	NP	?	NP-Complete	NP	NP	NP[CL97]
	non approx	$\exists\varepsilon > 0,\ k^{\varepsilon}$	$\exists\varepsilon > 0,\ k^{\varepsilon}$	?	———	?	$\forall\varepsilon > 0,\ \|V\|^{\frac{1}{4}-\varepsilon}$	$o(\log(\|V\|))$ [WCX02]
	approx	k	k	k	———	?	?	$O(\log(\|V\|))$ [WCX02]
span 1	complexity							NP[CL97]
	non approx	P [DS04]	P	P	P	P	P	$o(\log(\|V\|))$ [WCX02]
	approx							$O(\log(\|V\|))$ [WCX02]
bounded degree		———	P	P	———	———	———	———

? = open question, $\delta = (\log\log|\mathcal{C}|^{\frac{1}{2}})^{-\varepsilon}$ for $\varepsilon < \frac{1}{2}$, ——— = do not apply.

Table 1: Complexity and approximability properties of colored problems.

Finally, an efficient MILP formulation for the minimum color path problem has been given in [CPRV06].

It is worth noting that Theorem 3.1 might be very useful for the resolution of more general problems, typically for solving a multi-commodity flow problem subject to SRGs constraints, using a column generation formulation of an integer linear program.

3.1.2 Color disjoint *st*-path

The fundamental problem called the color disjoint paths problem as also been studied. It consists in finding two paths between a pair of nodes in the virtual network such that no single failure in the underlying layer may cause both paths to fail simultaneously. This problem is much more difficult than the traditional disjoint path problem of graph theory [CDP+07, Bha99] and recent studies have proven its NP-completeness [Hu03, SYR05]. In [CDP+07], it was shown that it is NP-hard to approximate the maximum number of disjoint paths in the SRRG settings: that is no polynomial time algorithm can provide solutions which are guaranteed to be lower than the optimal number of paths multiplied by some factor depending on the size of the graph. This fosters the relevance of the different heuristic approaches studied for this problem [YVJ05, TR04a, SYR05].

However, when all colors have span 1, the problem of determining two color disjoint *st*-paths can be solved in polynomial time. It has first been proved for the case where all edges of a same color have a common end-node[DS04], that is a particular case of colors of span 1, with some extensions in [LW05, LW06]. Then, the results has been generalized to colors of span 1 [CHPV07].

3.1.3 Trap topologies

One of the common problems that arise in restoration path computation is the existence of a trap topology [XXQ03]: if a service path is routed over a trap topology, then there may not exist a diverse restoration path, even though two diverse paths exist in the network. A challenge that SRRG protection schemes have to face consists in the impossibility to provide 100% reliability against certain multiple link failure events, depending on the SRRG configuration [YVJ05]. In these cases, an objective may be to find one or more paths for each connection, such that the reliability for each connection is maximized: it is the minimum overlapping paths problem. It consists in finding a set of paths sharing a minimum number of SRRG, or colors. This problem as well as the minimum cost SRRG diverse routing problem and the routing problem under both link capacity and path length constraints have also been shown to be NP-complete [Hu03].

3.1.4 Average reliability

Given a network and a set of requests, the problem of finding a routing minimizing the average number of SRG through which the route of a request goes has already been studied. It has been proved to be NP-hard [SYR05], hard to approximate [CDP+07], and heuristic algorithms have been designed. For example [SYR05] studied a *Tabu Search* heuristic algorithm for routing requests with different requirements.

However, if we consider a network with edge cost, such an objective may lead to a costly routing. To avoid this, the objective to be minimized has to take into account two cost criteria. First we want to minimize the average failure probability, that is the average number of risks of failure on which a connection is dependent, as previously. Secondly, since an operator is interested in both the safety of the connections and their operating costs, the objective has to take into account the cost of the edges used along the route of the connections. Hence, we concentrate on the problem of computing under capacity constraints a set of paths minimizing a linear combination of their edge costs and the number of SRGs through which they are routed.

An integer linear program (ILP) with a column generation formulation for the Minimum Average Color Flow problem has been proposed in [CHPV07].

3.2 Protection with cycles

We study the problem of designing a survivable WDM network based on covering the communication requests with subnetworks that are protected independently from each other. The subnetworks are chosen to be loops (cycles) in order to minimize the complexity of the routing problem with full survivability.The advantage is that a loop (cycle) is secured by its reverse loop. Given the failure of any single link, we can reroute the traffic going through the failed link via the other part of the cycle. (More precisely one can associate two wavelengths to each cycle of the covering: one for the normal traffic and another as a spare one). This problem was asked by France Telecom R & D

We model the physical communication network by a graph, called the physical graph and denoted by G. It is a symmetric digraph, but we will see that we only need to consider the underlying undirected graph. The family of communication requests (or an instance of communications) is modeled by another graph, called the logical (or virtual or request) graph and denoted by I. The vertex set of the logical graph is the same as that of the physical graph and the edges correspond to the requests between these vertices. We will suppose that the requests are symmetric. Therefore, the logical graph will be a symmetric digraph. Routing an instance consists of associating a directed path in the physical graph G to each request (arc of I).

Finally we suppose that the routing of symmetric requests is done by symmetric routing (that is the way done in backbone networks of telephone companies). The symmetry of the routing implies that we can consider undirected graphs instead of symmetric digraphs for both the physical and logical graphs. Therefore, routing an instance consists of associating an undirected path in the physical graph G to each (symmetric) request (edge of I). A routing is called a shortest path routing if each path in the routing is a shortest path in the physical graph G. The load of an edge of G is the number of paths of the routing which contain this edge.

The survivability problem mentioned above consists of finding a cycle partition or covering of the edges of I with an associated routing over the graph G which should satisfy the Disjoint Routing constraint, or *DR constraint*, i.e :the requests involved in a cycle of the covering are routed via vertex disjoint paths (equivalently, their routings form an elementary cycle in the physical graph G).

The aim is to minimize the cost of the network; that is a very complex function depending on the size of the ADM's put in each node, the number of wavelengths (associated to the subnetworks) in transit in each optical node and a cost of regeneration and amplification of the signal. In a first approximation, some authors reduce it to minimize the number of cycles of the covering (which is related to the problem of minimizing the number of wavelengths used and the cost of transmission); other minimize the sum of the number of vertices of the rings ; other insist on using very small cycles in the covering (short cycles are easier to manage and in case of failures, rerouting is easier). One can also want to minimize the total load (using or not shortest paths).

A routing is called *optimal* (resp. *quasi optimal*) if it satisfies the following conditions:

(a) all paths are shortest paths,

(b) the DR constraint is satisfied,

(c) the load for the edges of G is uniform (resp. quasi uniform).

Remark. In some cases, it is impossible to have the same load on all edges and this happens when the total number of edges in the paths of the routing is not divisible by the number of edges in the physical graph. In this case, a uniform load means that the difference between the maximum and minimum load is one and a quasi uniform load corresponds to a difference of two.

In summary, we are interested in finding a cycle partition of the edges of a logical graph I such that the routing associated with the requests is optimal or quasi optimal. We have obtained exact results when the logical graph I is K_n, which corresponds to the instance of communication called total exchange or all-to-all, where each vertex wants to communicate with all the others simultaneously, and when the the physical graph G is C_n, a cycle of length n [BCY03].

In our recent work [BY07] another particular case of the general problem is considered. We assume that the physical graph is a square torus $T(n)$, which can be considered as the Cartesian product of two C_n's, and the logical graph is the complete graph K_{n^2} corresponding to all-to-all communication. Notice that for n odd, as the degree of all vertices of K_{n^2} is even, a possible optimal solution is a cycle partition (instead of a cycle covering) which satisfies the requirements. For n even, as each degree in K_{n^2} is odd, at least $n^2/2$ edges (requests) have to be covered twice in any cycle covering of K_{n^2}. The best we can do is to have a cycle partition of $K_{n^2} + F$, where F is a 1-factor, and in this case, for minimizing the load, the edges of the 1-factor should be routed by paths of length one in $T(n)$. We indicate the results obtained

Theorem A. Let n be odd. The minimum size of a cycle partition of K_{n^2}, with an associated optimal routing over $T(n)$, is exactly $n(n^2-1)/4$.

Theorem B. Let $n = 2k$. There exists a cycle partition of $K_{n^2}+F$, where F is a 1-factor, of size $n^3/4+cn^2$ with an associated optimal routing over $T(n)$ when k is odd and a quasi optimal routing when k is even.

Remark: Theorem A gives an optimal solution, but Theorem B only gives a solution which is asymptotically optimal with respect to the size of the partition as a lower bound on the number of cycles is $(n^3+4)/4$.

Theorem C. Let n be odd. There exists a $\{C_3, C_4\}$-partition of K_{n^2} with an associated optimal routing over $T(n)$.

Theorem D. Let $n = 2k$. There exists a $\{C_3, C_4\}$-partition of $K_{n^2}+F$, where F is a 1-factor, with an associated optimal routing over $T(n)$ when k is odd and a quasi optimal routing when k is even.

3.3 Network Reconfiguration

We consider multifiber WDM networks. The virtual topology is constituted of lightpaths obeying the wavelength continuity constraints. The traffic follows a all-to-all pattern. The traffic evolutions are discretized.

We deal with the *network reconfiguration problem*. It is an extension of the static *Routing and Wavelength Assignment* (RWA) problem, which is proven to be a NP-hard. Hence the reconfiguration problem is also NP-hard. Remember that the static RWA problem can be stated as, given a network and a traffic matrix, we need to determine the logical topology to be imposed on the physical topology, hence routing the lightpaths over the physical topology and assigning a wavelength to each lightpath. At the same time the RWA problem is solved, it is common to route the packet traffic over the logical topology obtained.

In the reconfiguration problem, we consider a physical topology and a succession of traffic matrices. For each traffic matrix, we solve the associated RWA problem and we route the packet traffic, obtaining a succession of virtual topologies and packet routings. The main difference between a succession of RWA problems and the reconfiguration problem is that the latter takes into consideration the fact it will be required to switch from a virtual topology and the associated routing to the next one. Such operation may generate network disruption, which is not desired. When two virtual topologies are similar, it is easy to switch from one to the other. The reconfiguration problem involves a trade-off between the virtual topology quality and the network disruption that may occur each time it is re-defined.

The network reconfiguration problem is well known in literature. However, there are not too many satisfying methods to solve it. Some works restrict themselves to very-specific cases. In [NTM00], the authors develop reconfiguration algorithm for ring networks. The proposed algorithm is based on branch-exchange techniques. In [BR01] a Markovian process is used to study the trade-offs involved in reconfiguration in single-hop broadcast WDM networks. The work in [BM00a] considers the reconfiguration problem in the case of a unique traffic evolution, and not as a succession of traffic evolutions, and use as input an existing virtual topology and routing.

Different Mixed Integer Linear Programming (MILP) approaches has been developed in literature which addresses short-term, mid-term and long-term network reconfiguration issues with respect to evolving traffic in WDM optical networks (see [SSS02].

The problem has been also addressed in [ZZZM02, JPM03] under the context of "dynamic traffic grooming". In both works, the authors modify the initial network graph. The modifications consist of splitting nodes to represent different part of the optical devices (electronic processing, purely optical router, and so on). That allows to use quite simple algorithms based on the shortest path [ZZZM02] solving the problem with elegant mathematical models [JPM03]. However, these works focus on the grooming aspect and does not consider the adaptation of the virtual topology to meet the ever increasing traffic demands across multiple periods of network evolution.

Our approach in citeHD07 takes into account the trade-off between the network configuration quality and the network disruption, which is generally absent in the articles found in the literature. Since this trade-off is the essence of the reconfiguration problem, we believe it cannot be left aside. The practical objective of this work is to make the best virtual topology reconfigurations in relation to predicted traffic evolution.

We present a *Mixed Integer Linear Programming* (MILP) formulation to solve this problem. However, the MILP approach is unable to solve large network instances within reasonable time limits. Meta-heuristics, such as the *simulated annealing* (SA), are generally able to find good solutions to optimization problems for an affordable computational cost [DSS03]. As we compare the results obtained with a MILP approach and a SA approach, we can use the information provided by the solver to make a pertinent evaluation of the SA performance.

4 Resource scheduling in wireless networks

4.1 Fair time sharing in wireless LANs

Wireless local area networks are often based on the famous IEEE 802.11 technology to communicate on the wireless medium [oEI97]. If this technology is rather simple to use, it also leads to several issues. One of these problems is known as the performance anomaly of 802.11. This issue comes from the possibility to have multiple rates in a 802.11 cell. Indeed, with 802.11b for instance, each station can choose between four transmissions rates (1, 2, 5.5 and 11 Mbps). Heusse *et al.* [HRBSD03] have shown that the presence of slow stations in a multi-rate wireless network slows down all other stations. During the transmission of a slow station the medium is busy for a longer period than during the transmission of a fast station, assuming the same packet size. Since IEEE 802.11 provides simple per-packet fairness in a single-hop network, this means that in a long period each emitter has statistically sent the same number of frames. On a time basis, however, slow stations have occupied the channel for a longer period of time. This time unfairness leads to a loss of performance due to the existence of slow transmissions.

Several solutions have been proposed in the literature to solve this problem. Some of them are based on a static predefined time sharing between slow and fast stations, by shaping the MTU (Maximum Transmission Unit) on a transmission rate basis. Other approaches set a maximum amount of time a station can hold the medium, like with the TXOP (transmit opportunity) introduced in the IEEE 802.11e standard. Finally, other approaches try to adapt the contention window size of IEEE 802.11, according to the transmission rate of the station. The main problem of existing solutions is that they are static or centralized.

In [RGLIF07], we tackle both issues, solving the performance anomaly with a dynamic and distributed approach. Our solution is dynamic because it introduces a transmission time, as the TXOP, that changes depending on the perceived channel occupancy, which evolves with the traffic load of the network. Our solution is a distributed approach because each node computes locally the maximal channel occupancy time. The carrier sensing mechanism provided by IEEE 802.11 natively allows this computation. Once a node gains access to the medium, it can send a burst of packets. The number of transmitted packets is limited by the computed transmission time, which depends on the maximal occupancy time perceived by the station.

The idea of our protocol, called PAS (Performance Anomaly Solution), is based on the fact that each station should have the same transmission time on the radio channel. Therefore, if an emitter senses a channel occupancy time that is longer than the transmission time of the current packet to be emitted, then it can aggregate more packets in order to get a better channel occupancy time. The aggregation is realized by spacing the reception of the previous packet's acknowledgment and the emission of the next packet with a SIFS. There are two main mechanisms in PAS: the first one is the medium sensing mechanism that computes the transmission time; the second one is the packets' sending, based on the transmission time computed previously.

The first mechanism for the computation of the allowed transmission time is given in Algorithm 1. A station always senses the radio medium, and maintains the channel occupancy time. This time is the channel busy time due to a transmission, including transmission that can be only sensed but not decoded (*i.e.* in the carrier sensing area). Each station maintains the maximum channel occupancy time in a variable called t_p_max. This parameter is set to 0 after each successful transmission of the station. This avoids the station from monopolizing the channel after a transmission and improves the reactivity of the protocol. Furthermore, this mechanism reduces the short time unfairness that can arise when the same node successively accesses the radio channel.

Algorithm 1 Performance Anomaly Solution - Sensing Phase

```
t_p_max := 0;
repeat
   if (a signal is sensed at the physical layer) then
      t_p_current := signal's channel occupancy time;
      if (t_p_current > t_p_max) then
         t_p_max := t_p_current;
      if (packet type == ACK) and (Destination == me)  then
         t_p_max := 0;
until 1;
```

The second mechanism concerns the emission phase and is given in Algorithm 2. The station can either transmit its packet classically by using the medium access mode of IEEE 802.11 or aggregate some of its packets. To know whether it can aggregate or not, it uses the parameter t_p_max: if its channel occupancy time is smaller than the value of this variable, then it can aggregate. In Algorithm 2, t_my_packet is the time required to send the current packet, while t_my_left corresponds to the remaining allowed transmission time. The value of this last parameter evolves with time and with the packets previously emitted. When this value becomes too small, no more aggregation is possible. Otherwise the medium occupancy time of this station would become larger than the maximum transmission time sensed on the channel, which is not fair.

Algorithm 2 Performance Anomaly Solution - Emission Phase

```
sending := false;
t_my_left := 0;
for (each packet to send) do
   if (t_my_left ≤ 0) then
      t_my_left := t_p_max;
   α = (⌈t_my_left / t_my_packet⌉ − t_my_left / t_my_packet) ∗ t_my_packet;
   t_my_left := t_my_left − t_my_packet;
   if (sending == true) then
      if (t_my_left + α > 0) then
         aggregated_sending();
      else
         t_my_left := 0;
         sending := false;
         classical_sending();
   else
      if (t_my_left + α > 0) then
         sending := true;
         classical_sending();
      else
         t_my_left := 0;
         classical_sending();
```

In [RGLIF07], we have shown, through both analytical analysis and simulation, that PAS solves the performance anomaly in many scenarios for both UDP and TCP traffic.

The aggregate throughput can be increased and the time-based fairness is almost reached in nearly every of the tested configurations.

4.2 Multi-objective analysis in wireless mesh networks

There is an increasing interest in using *Wireless Mesh Networks* (*WMNs*) as broadband backbone for next-generation wireless networking. We consider the *Round Weighting Problem (RWP)*. It solves a joint routing and scheduling problem to attend a given demand subjected to the multiaccess interferences. We propose a multiobjective approach for the *RWP*, considering two objective functions. A *column generation* approach is used to select the rounds improving the objective function, reducing the complexity in generating the whole set of rounds which is exponential. We make experiments on networks with different numbers of sinks. Our approach captures the trade-off generated by using these two conflicting objective functions. This relationship corresponds to a convex piecewise linear function.

The Round Weighting Problem was treated in [KMP04] with the objective to minimize the rounds number (time). The authors make dual analysis and propose approximation algorithms for some specific graphs. They showed that this problem is NP-hard by proving that the well-known NP-hard problem of finding the *Fractional Coloring* on unit graphs reduces to it. In [GMRR07] the same problem with gateways placement was showed very hard even for small networks, the program generates MILPs with thousands of constraints and integer variables. Therefore, large problem instances (> 10 nodes) cannot be solved to optimality and only approximate solution can be obtained. A column generation approach has been developed for an efficient generation of feasible rounds in [ZWZL05]. Here we described the work done in [GH07].

4.2.1 Hypotheses and problem definition

In this section we give some definitions that will help to understand the problem. The *RWP* can be modeled as a graph problem. A wireless topology is represented as a digraph $G = (V, E)$. The set of routers is represented by the set of nodes V. The set of edges $E \subseteq V \times V$ corresponds to the communication links from the real network. If a router j is located within the transmission range tr_i of a router i, considering range distance, obstacles, etc, then there is an edge $(i, j) \in E$.

We consider the link (i, j) active when the router i is transmitting data to j. In this case, it interferes with another links located within the interference range it_i of router i. The set $I_{u,v}$ is composed by all links interfering with the link (u, v). Consequently flexible binary interference models can be adopted.

A round in a wireless network corresponds to a set of links that can be active at the same time without interferences among them. The size of the complete set of rounds is exponential. We consider a column generation approach to select as required the rounds to improve the solution of the problem. The round definition guarantees that the communication will be multiaccess interferences free in G.

We focus on router-gateway traffic pattern, naturally modeled by a multicommodity flow problem. The commodities are going from the set of nodes V_r to the set of gateways V_g ($V_r \cup V_g = V$ and $V_r \cap V_g = \emptyset$).

Given a graph $G(V_r \cup V_g, E)$, a set of router demands d_v with $v \in V_r$ and an interference model, the *Round Weighting Problem (RWP)* is to find the set of rounds to satisfy the given demand. From this set of rounds can be deduced the paths followed by the data. We deal with two objectives: *MinMaxLoad* and *MinTime*. In *MinMaxLoad* we try to balance the load in the routers and in *MinTime* the goal is to minimize the communication time.

4.2.2 Mathematical formulation

We define the following variables: Let the variable $x^v_{i,j}$ denotes the flow from the router v over link i, j. The demand from each router v is represented by the parameter d_v. Let the binary parameter $a^r_{i,j}$ be 1 if link (i, j) is active in the round r, and 0 otherwise.

Recall that set $I_{u,v}$ is composed by all links interfering with (u, v). We define $F^{(i,j)}_{(u,v)} = 0$ if $(i, j) \in I_{u,v}$ and 1, otherwise. We define w_r as the fraction of time that round $r \in R$ is scheduled. Consequently, there is an induced edges capacity $c_{i,j} = \sum_{r \in R} a^r_{i,j} w_r, \forall\, (i, j) \in E$.

The master problem can be defined as follow: Given a graph $G(V_r \cup V_g, E)$, a set of routers demand d_v with $v \in V_r$ and a set of rounds R, the problem is to assign a weight w_r to each round $r \in R$. The weights represent the amount of time a round will be activated. The total amount of time needed to satisfy all demand will be $w_R = \sum_{r \in R} w(r)$. From the edges of the rounds can be deduced the paths followed by the data. It may happen that some of the rounds r have a weight equal to zero. The load in each router $i \in V_r$ is given by $l_i = \sum_{v \in V_r} \sum_{j \in V/(i,j) \in E} x^v_{i,j}$. The constraints of the master problem expressed as a linear programming model are the following:

$$\sum_{i \in V_/(v,i) \in E} x^v_{v,i} = d_v, \forall v \in V_r \quad (1)$$

$$\sum_{j \in V_g} \sum_{i \in V_r/(i,j) \in E} x^v_{i,j} = d_v, \forall v \in V_r \quad (2)$$

$$\sum_{i \in V_r/(i,j) \in E} x^v_{i,j} - \sum_{k \in V/(j,k) \in E} x^v_{j,k} = 0, \forall j, v \in V_r \quad (3)$$

$$\sum_{r \in R} a^r_{i,j} w_r - \sum_{v \in V_r} x^v_{i,j} \geqslant 0, \forall i, j \in E \quad (4)$$

To express the subproblem as a linear programming model, we have to define some additional notations. Let $p_{(i,j)}$ be the dual variable for the link demand constraint 4 in the master problem. Consider the binary variable $u_{(i,j)}$ indicating if the edge (i, j) enters the round to be added to R. The subproblem model is the following:

$$\min \left(1 - \sum_{(i,j) \in E} p_{(i,j)} u_{(i,j)} \right) \tag{5}$$

$$u_{(i,j)} + u_{(k,l)} \leqslant 1 + F_{(i,j)}^{(k,l)}, \forall (i, j) \in E, \forall (k, l) \in E \tag{6}$$

If the value of the objective function in the subproblem is negative, a new column can be identified $u_{(i,j)}$ and the master basis is expanded. Otherwise, the master problem already gives the optimal solution to the original problem.

4.2.3 Multiobjective Formulation

To evaluate the overall quality of our solutions, we can use the following metrics:

- *MinMaxLoad* (f^1): Minimizing the quantity of flow over the nodes. The rounds are chosen in a way to minimize the maximum load l_v in the router nodes V_r.
- *MinTime* (f^2): Minimizing the time of the communication. It chooses the rounds in a way that the round activations time will be minimum. The goal is to minimize the total weight w_R of the schedule. The weight w_r define how many times the round r will be activated and it can be mapped to time-slots to rounds (edges) activation in G.

The objective function of the master problem with objective *MinMaxLoad* and *MinTime* are, respectively, the following

$$\min(f^1 = \max_{v \in V_r}(l_v)) \tag{7}$$

$$\min(f^2 = w_R) \tag{8}$$

In multiobjective optimization, the solution space is a part of R^m where m is the number of metrics. In our case, as we consider two objectives, we have $m = 2$. The optimization is performed on the plane, and as there is no total order relation in R^2, there is not a single but many "best solutions", forming a region called *Pareto optimal set* [Par96].

4.2.4 Results

Practical results have computed using the mesh network instances given in [OPTW07]. We represented some of the obtained results on figure 7 and figure 8. The results obtained are represented in the solution space. The x axis represents the communication time, and on the y axis represents the maximum load. Each point corresponds to a solution.

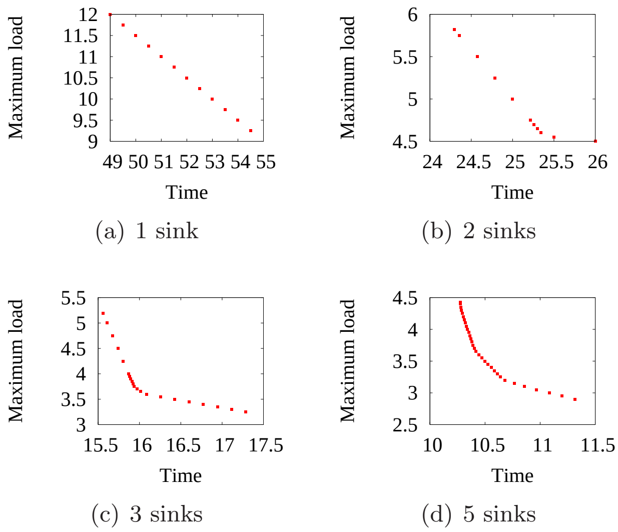

Figure 7: 39 nodes mesh network (giul69 instance)

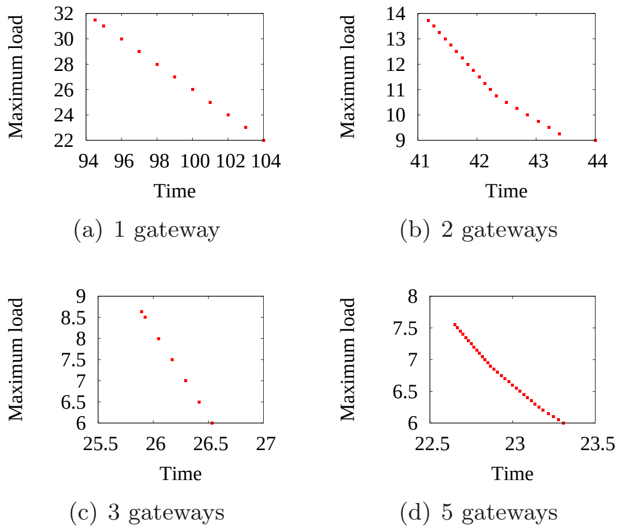

Figure 8: 65 nodes mesh network (ta2_65 instance)

This approach, using column generation and multiobjective optimization, appears to be quite efficient: the computation time to solve any instance is low, of the order of tenths of seconds. The overall time f^1 as well as the maximum load f^2 decrease as the number of gateways increases.

As expected, the routing generates bottlenecks located around the gateway(s) because all the flow goes toward them. We observe that when the routing use distinct paths to route the flow, it allows to activate different edges in the same round, reducing the overall transmission time. Informally speaking, it may be more efficient to follow different routes that do not interfere one with another than following shorter routes resulting in more interferences.

Minimizing the time increases the maximum load of the routers. We observe that the relation between the maximum load and the transmission time seems convex and piecewise linear. The linear parts corresponds to the following situations: As we make tighter the value of the maximum load, an amount of flow is deported on another path. Using this other path results in an increase of the overall transmission time. Hence, for each unit of flow following the second path, the overall transmission time increases by a given value (the difference of time between the first path and the second one).

Each disruption in the graphs is due to the happening of a new bottleneck, forcing a flow transfer on a path that is not the best possibility. It may activate some path that was not in use. As a consequence, the rate of time per flow increases.

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